ELSEVIER

SCIENCE @ DIRECT®

PHYSICS LETTERS B

Physics Letters B 587 (2004) 117–120

www.elsevier.com/locate/physletb

A comment on the self-tuning of cosmological constant with deficit angle on a sphere

Hyun Min Lee

Physikalisches Institut der Universität Bonn, Nussallee 12, D-53115 Bonn, Germany

Received 10 September 2003; accepted 2 March 2004

Editor: G.F. Giudice

Abstract

In the 6D brane world model with a 4-form flux on a sphere S^2 for self-tuning the cosmological constant, we comment on the fine-tuning problem in view of the quantization of the dual 2-form flux and the orbifolding case S^2/Z_2.

PACS: 98.80.Es; 98.80.C; 12.25.Mj

Keywords: Cosmological constant; Self-tuning; Brane

For recent few years, the extra-dimensional models where the Standard Model (SM) fields reside in the hypersurface (so-called the brane) of a higher-dimensional spacetime have drawn much attention, in particular, with the hope of solving the cosmological constant problem. In these brane world models, there is a possibility that the SM quantum corrections, which contributes only to a brane tension, could be under control by some bulk relaxation mechanism. We are interested in the *self-tuning* solution in the sense that a 4D flat solution is always obtained by choosing an integration constant of the bulk solution without a fine-tuning between Lagrangian parameters [1].

In 5D models, it has been shown that the attempt with a bulk massless scalar has a hidden fine-tuning in curing the naked singularity of the warp factor [2] and some self-tuning solutions need a particular type of the bulk action [3] or higher curvature terms [4]. In any case, in 5D self-tuning models, changing a brane tension needs a change of the bulk solution.

On the other hand, in a 6D model with extra dimensions compactified on a factorizable S^2 with a deficit angle, it has been known that changing a brane tension needs only changing a deficit angle once there is a bulk tuning through the flux [5–9]. In this case, the flux is also responsible for the stabilization of extra dimensions. For a concrete example, the bulk 2-form flux has been considered [5,7–9] but it has been shown that the quantization condition makes the flux dependent on the deficit angle and thus a fine-tuning between bulk and brane cosmological constants is indispensable [8,9]. For other example, however, when the bulk 4-form flux is considered instead of the 2-form flux, it has been claimed that there is no flux quantization or no deficit angle dependence of the 4-form flux [9]. Recently, it has been shown that this 6D self-tuning model with 2-form or 4-form flux is not

E-mail address: minlee@th.physik.uni-bonn.de (H.M. Lee).

0370-2693 © 2004 Published by Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2004.03.008

different from the old 4D one with a *non-dynamical* 4-form flux from the 4D effective theory point of view [10].

In this Letter, we consider a self-tuning solution with a 3-form field A_{MNP} $(M, N, P = 0, 1, 2, 3, 5, 6)$ in 6D with extra dimensions compactified on a two-sphere S^2 with a deficit angle [9]. We show explicitly that there appears a quantization condition with the deficit angle dependence when the 3-form field couples to a magnetic source. We comment more on the fine-tuning problem in this model.

On any six-dimensional manifold, the dual transformation of the field strength $H_{MNPQ} = \partial_{[M}A_{NPQ]}$ into a 2-form field strength $F_{MN} = \partial_{[M}A_{N]}$ is defined as [11]

$$F_{MN} = \frac{1}{4!}\sqrt{-g_6}\epsilon_{MNPQRS}H^{PQRS}, \tag{1}$$

where g_6 is the 6D metric determinant and ϵ_{MNPQRS} is the Levi-Civita symbol with $\epsilon_{012356} = -\epsilon^{012356} = 1$. Then, the 6D action with a 3-form field and its coupling to both electric and magnetic sources is

$$\begin{aligned} S = \int d^4x\, d^2y\,\sqrt{-g_6} \\ &\times\left(\frac{M^4}{2}R - \Lambda_b - \sum_{i=1}^{2}\frac{\sqrt{-g_4^{(i)}}}{\sqrt{-g_6}}\Lambda_i\delta^2(y-y_i)\right) \\ &- \frac{1}{2\cdot 4!}\int H\wedge *H - e\int_{W_3} A_3 - g\int_{W_1} A_1, \end{aligned} \tag{2}$$

where $H = dA_3$, and $*H$ is the Hodge dual of H, and A_1 comes from the definition of the dual field strength $F = dA_1$, and W_3, W_1 denotes the world volumes of electric and magnetic sources, respectively. Here, M is the 6D fundamental scale, $g_4^{(i)}$ are the 4D metric determinants, Λ_b, Λ_i are bulk and 3-brane cosmological constants and e, g are electric and magnetic charges of sources for the 3-form field.

Now let us take the metric ansatz as the direct product of 4D space and a two-sphere with a deficit angle $2\pi(1-\beta)$,

$$ds^2 = g_{\mu\nu}(x)\,dx^\mu\,dx^\nu + \gamma_{mn}(y)\,dy^m\,dy^n, \tag{3}$$

with

$$\gamma_{mn}(y)\,dy^m\,dy^n = R_0^2\big(d\theta^2 + \beta^2\sin^2\theta\,d\phi^2\big), \tag{4}$$

and the ansatz for the field strength H as

$$H_{\mu\nu\rho\sigma} = \sqrt{-g}\epsilon_{\mu\nu\rho\sigma}E, \qquad \text{others} = 0, \tag{5}$$

where E is an arbitrary constant. Then, the ansatz for H satisfies both the field equation and the Bianchi identity for H

$$\partial_M\big(\sqrt{-g_6}H^{MNPQ}\big) = 0, \qquad \partial_{[M}H_{NPQR]} = 0. \tag{6}$$

Moreover, the Einstein equation to be also satisfied is

$$G_{MN} \equiv R_{MN} - \frac{1}{2}Rg_{MN} = \frac{1}{M^4}T_{MN}, \tag{7}$$

where

$$\begin{aligned} T_{MN} = &-\begin{pmatrix} \big(\Lambda_b + \frac{1}{2}E^2\big)g_{\mu\nu} & 0 \\ 0 & \big(\Lambda_b - \frac{1}{2}E^2\big)\gamma_{mn} \end{pmatrix} \\ &- \sum_{i=1}^{2}\frac{\Lambda_i}{\sqrt{\gamma}}\begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix}\delta^2(y-y_i). \end{aligned} \tag{8}$$

Here, the non-vanishing components of the Einstein tensor [5] are given by

$$G_{\mu\nu} = (R_4)_{\mu\nu} - \frac{1}{2}(R_4 + R_2)g_{\mu\nu}, \tag{9}$$

$$G_{mn} = (R_2)_{mn} - \frac{1}{2}(R_4 + R_2)\gamma_{mn}, \tag{10}$$

where R_4 $((R_4)_{\mu\nu})$, R_2 $((R_2)_{mn})$ are the Ricci scalars (tensors) for the 4D space and the two-sphere, respectively.

Then, for a 4D flat solution with $(R_4)_{\mu\nu} = 0$ and $R_4 = 0$, the bulk equation gives two conditions

$$E^2 = 2\Lambda_b, \tag{11}$$

$$R_0^{-2} = M^{-4}\left(\Lambda_b + \frac{1}{2}E^2\right), \tag{12}$$

while the boundary conditions at the branes determine the deficit angle in terms of brane tensions

$$2\pi(1-\beta) = \Lambda_1 = \Lambda_2. \tag{13}$$

At first sight, it seems that there always exists a flat solution for arbitrary brane tensions once the 3-form flux takes a value satisfying Eq. (11). Since the field strength of the 3-form takes a value only along the 4D space, it looks independent of the geometry of extra dimensions such as the deficit angle. Moreover, it was argued that the fine-tuning between two brane tensions in Eq. (13) is avoidable by considering S^2/Z_2 instead of S^2 [6,9]. As in the case at hand, since there is no warp factor in the 4D metric part, it seems that one does not need to introduce a 4-brane in the action even after Z_2 orbifolding.

However, this is just the result of disregarding the coupling of the 3-form field to the magnetic sources. By duality of Eq. (1), the ansatz for the 4-form field strength of Eq. (5) becomes the ansatz for the 2-form field strength in the spherical coordinate of extra dimensions

$$F_{\theta\phi} = -\sqrt{\gamma}\epsilon_{\theta\phi}E, \qquad \text{others} = 0. \tag{14}$$

Then, with the metric (3), the 1-form gauge field solution is obtained for the upper and lower hemispheres as

$$A_{\phi,\pm} = \beta E R_0^2(\cos\theta \mp c), \tag{15}$$

where $c = 1$ from the identity, $\int_{S^2} F_2 = \int_{S^1}(A_{1,+} - A_{1,-})$. Then, the solutions of the gauge field must be related by a gauge transformation [8,9]:

$$A_{\phi,+} = A_{\phi,-} + \partial_\phi\alpha(\phi), \tag{16}$$

where $\alpha(\phi) = -2\beta E R_0^2\phi$. Consequently, from the single-valuedness of the gauge transformation $e^{ig\alpha(\phi)}$, we get the quantization condition

$$E = \frac{n}{2g\beta R_0^2}, \tag{17}$$

with n integer. Then, using this quantization condition and Eq. (12), the bulk fine-tuning condition (11) becomes

$$\frac{n^2}{2g^2\beta^2} = \frac{M^8}{\Lambda_b}. \tag{18}$$

Thus, we find that from the quantization of the dual field, the brane tension also enters the fine-tuning condition via the deficit angle.

Now let us remark on the possibility with S^2/Z_2. In this case, when we consider the covariant derivative for a charged field under the 1-form gauge field, the gauge field transforms under the Z_2 reflection, $\theta \to \pi - \theta$ and $\phi \to \phi$, as

$$A_\theta \to -A_\theta, \qquad A_\phi \to A_\phi. \tag{19}$$

Thus, the field strength also transforms under Z_2 as

$$F_{\theta\phi} \to -F_{\theta\phi}. \tag{20}$$

Then, we get the field equation for F as

$$\partial_\theta\left(\sqrt{\gamma}F^{\theta\phi}\right) = E\delta\left(\theta - \frac{\pi}{2}\right) \neq 0. \tag{21}$$

This means that in order to match the boundary condition for F, we must introduce around the equator an extended (4-brane) *electric* source under A_1, which is an extended *magnetic* source under A_3. Then, the 6D action for the dual 2-form becomes

$$\int d^4x\, d^2y\, \sqrt{-g_6} \times \left(-\frac{1}{4}F_{MN}F^{MN} - \frac{\sqrt{-g_5}}{\sqrt{-g_6}}A_aJ^a\delta\left(\theta - \frac{\pi}{2}\right)\right), \tag{22}$$

where a runs over $0, 1, 2, 3, 6$, and g_5 is the determinant of the induced 5D metric on the equator with $ds_5^2 = ds_4^2 + R_0^2\beta^2\, d\phi^2$, and $J^a = Q\delta^a_\phi$ with an electric charge Q under A_1. Therefore, the modified field equation for A_1 determines the dual 2-form flux in terms of the charge Q as

$$E = R_0\beta Q. \tag{23}$$

Using Eqs. (11) and (12), this result leads to a necessary condition for the charge as

$$Q = \pm\frac{E}{R_0\beta} = \pm\frac{2\Lambda_b}{\beta M^2}. \tag{24}$$

Due to the Z_2 property of $F_{\theta\phi}$ (20), the general solution for A_ϕ for the upper and lower hemispheres is given by

$$A_{\phi,\pm} = \pm\beta E R_0^2(\cos\theta + c_\pm), \tag{25}$$

where $c_\pm$ are integration constants with $c_+ = -c_-$ for the Z_2 even A_ϕ. The 4-brane source term generically contributes to the energy–momentum tensor as

$$T_{MN} = \left[\frac{1}{2}(A_aJ_b + A_bJ_a) - g_{ab}A_cJ^c\right] \times \frac{\sqrt{-g_5}}{\sqrt{-g_6}}\delta^a_M\delta^b_N\delta\left(\theta - \frac{\pi}{2}\right), \tag{26}$$

which becomes under our solution

$$T^0_0 = T^i_i = -\frac{Q}{R_0}A_\phi\delta\left(\theta - \frac{\pi}{2}\right), \qquad \text{others} = 0. \tag{27}$$

In order for the charged 4-brane not to contribute to the energy–momentum tensor, we need to have

$c_+ = 0$ on the equator as the boundary condition for the gauge field. However, the Stokes theorem does not hold around the 3-brane, i.e., $\int_\Sigma F_2 \neq \int_{\partial\Sigma} A_1$ where Σ is the infinitesimal surface surrounding the 3-brane. Then, there are two probable solutions for this: one is to modify the field strength F, and the other is to choose a difference gauge choice with $c_+ = -1$ at the 3-brane.[1]

In the former case, the solution for $F_{\theta\phi}$ is supposed to be modified with a singular part,

$$F^s_{\theta\phi} = 2\pi\beta E R_0^2 \epsilon_{\theta\phi}\delta^2(y). \tag{28}$$

Then, the field equation for F implies that a charge on the 3-brane might be added with the 4D action [12]

$$-q\int d^4x\,\sqrt{-g_4}F_{\theta\phi}\epsilon^{\theta\phi}, \tag{29}$$

with $q = -2\pi\beta E R_0^2$. However, in this case, it would be indispensable to have the original solution modified with a $\delta^2(0)$ term coming from the singular part of $F_{\theta\phi}$ in the energy–momentum tensor.

On the other hand, in the latter case, where there are different gauge choices with $c_+ = -1$ and $c_+ = 0$, we can regard the gauge fields to be related to each other by a gauge transformation. Therefore, the dual 2-form flux on S^2/Z_2 is quantized as

$$E_{S^2/Z_2} = \frac{n}{g\beta R_0^2} = 2E_{S^2}, \tag{30}$$

where n is an integer and the subscript of E denotes the case we consider. Since the fundamental region on S^2/Z_2 is reduced to one hemisphere, it is reasonable to have the magnitude of the flux doubled, compared with the S^2 case. In this case, even if the original metric solution is maintained, there appears again a fine-tuning condition between brane and bulk cosmological constants in view of the flux quantization (30) as in the S^2 case.

We considered the 6D model with a 4-form flux where the self-tuning idea may be realized via the deficit angle on S^2. In this case, we have shown that the coupling of the 3-form field to magnetic sources is important for the self-tuning issue. First we have found that there appears a fine-tuning condition via the quantization of the dual 2-form flux. We also commented on the case with S^2/Z_2 for avoiding a fine-tuning between 3-branes. In this case, we showed that a 4-brane charge must be added on the equator due to the Z_2 property of the gauge field and the dual 2-form flux is also quantized with a deficit angle dependence for maintaining the original metric solution.

[1] The author thanks G. Tasinato for valuable discussion on this issue.

Acknowledgements

I would like to thank S. Förste, H.P. Nilles, A. Papazoglou and G. Tasinato for helpful discussions. I am supported by the European Community's Human Potential Programme under contracts HPRN-CT-2000-00131 Quantum Spacetime, HPRN-CT-2000-00148 Physics Across the Present Energy Frontier and HPRN-CT-2000-00152 Supersymmetry and the Early Universe. I am also supported by priority grant 1096 of the Deutsche Forschungsgemeinschaft.

References

[1] S. Kachru, M.B. Schulz, E. Silverstein, Phys. Rev. D 62 (2000) 045021, hep-th/0001206;
N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, R. Sundrum, Phys. Lett. B 480 (2000) 193, hep-th/0001197.
[2] S. Förste, Z. Lalak, S. Lavignac, H.P. Nilles, Phys. Lett. B 481 (2000) 360, hep-th/0002164;
S. Förste, Z. Lalak, S. Lavignac, H.P. Nilles, JHEP 0009 (2000) 034, hep-th/0006139.
[3] J.E. Kim, B. Kyae, H.M. Lee, Phys. Rev. Lett. 86 (2001) 4223, hep-th/0011118;
J.E. Kim, B. Kyae, H.M. Lee, Nucl. Phys. B 613 (2001) 306, hep-th/0101027.
[4] A. Jakobek, K.A. Meissner, M. Olechowski, Nucl. Phys. B 645 (2002) 217, hep-th/0206254.
[5] S.M. Carroll, M.M. Guica, hep-th/0302067.
[6] I. Navarro, hep-th/0302129.
[7] J.M. Cline, J. Descheneau, M. Giovannini, J. Vinet, hep-th/0304147.
[8] Y. Aghababaie, C.P. Burgess, S.L. Parameswaran, F. Quevedo, hep-th/0304256.
[9] I. Navarro, Class. Quantum Grav. 20 (2003) 3603, hep-th/0305014.
[10] H.P. Nilles, A. Papazoglou, G. Tasinato, hep-th/0309042.
[11] M. Nakahara, Geometry, Topology and Physics, in: Graduate Student Series in Physics, Hilger, Bristol, 1990.
[12] Y. Aghababaie, C.P. Burgess, J.M. Cline, H. Firouzjahi, S.L. Parameswaran, F. Quevedo, G. Tasinato, I. Zavala, hep-th/0308064.